Modelling the Galactic Magnetic Field using WMAP data at 22 GHz.

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ABSTRACT

We have studied the spatial structure of the three dimensional large-scale structure of the Galactic Magnetic Field using the large-scale polarization angle produced by the synchrotron emission at 22 GHz observed by WMAP. By using different models of the large-scale magnetic field of the Milky Way defined by different parameters it is found the best fit reproducing the direction of the polarization angle at 22 GHz by determining the Maximum Likelihood ( ) and obtaining the marginal probability distribution function for the different parameters. Once the parameters for each model are determined we apply again the Maximum Likelihood to determine the model that better describes the Galactic Magnetic Field.

Models of Galactic Magnetic Field

Since the discovery of the polarized synchrotron emission from our galaxy, several catalogues of rotation measure of radio sources (EGRS and pulsars) have been presented to constrain the model of Galactic Magnetic Field (hereinafter, GMF) of the Milky Way.
To study the observed GMF it is needed an equation describing the relativistic electron density distribution. The model used here is that used by Drimmel and Spergel (2001):

$$N_e = N_0 \exp\left(\frac{-r}{5kpc}\right) sech^2\left(\frac{z}{1kpc}\right)$$

with $N_0 \approx 10^{-13} cm^{-3}$, the value of the central density of relativistic electrons, and (r,z), the radial and the vertical coordinates in cylindrical galacto-centric coordinate system.
The expressions in cylindrical coordinates of the GMF model are given by different authors:

- **Model used by Page et al. (2007) :**

$$B_r = B_0 \cos\psi(r)\cos\chi(z) \qquad \psi(r) = \psi_0 + \psi_1 ln\left(\frac{r}{8kpc}\right)$$
$$B_\phi = B_0 \sin\psi(r)\cos\chi(z)$$
$$B_z = B_0 \sin\chi(z) \qquad \chi(z) = \chi_0 tanh\left(\frac{z}{1kpc}\right)$$

The parameter space for this model is constituted by [ψ_0, ψ_1, χ_0]. It is assumed B_0 ~ 3 μG

- **4-arms bisymmetric model (Han and Qiao, 1994):**

$$B_r = B_0(r)\cos\left(\theta - \beta \ln\left(\frac{r}{R_0}\right)\right)\sin(p)$$
$$B_\phi = B_0(r)\cos\left(\theta - \beta \ln\left(\frac{r}{R_0}\right)\right)\cos(p)$$
$$B_z = 0$$

The parameter space for this model is: [p, B_0], being B_0 a constant value with no dependence with the radius.

- **Axisymmetric model:**

$$B_r = B_0(r)\sin(p)$$
$$B_\phi = B_0(r)\cos(p)$$
$$B_z = k$$

The parameter space for this model is: [p, B0], being B0 a constant value with no dependence with the radius.

- **Concentric Circular Ring Model (Rand and Kulkarni, 1989):**

$$B_r = 0$$
$$B_\phi = B_0 \sin((\frac{\pi}{w})(r - R_0 + D_r))$$
$$B_z = constant$$

The parameter space is contituted by [D_r, w, B_0].

- **Turbulent Diffusion Model (Battaner and Florido, 2000):**

$$B_r = 0$$
$$B_\phi = K \arctan\left(\frac{z}{\sigma_1}\right)\exp\left(\frac{-z^2}{2\sigma_2^2}\right)$$
$$B_z = constant$$

The parameter space for this model is: [B0,σ_1, σ_2].

Analysis of the data at 22 GHz

For every considered model of GMF we obtain the direction of the polarization angle by using the equation:

$$\gamma(\hat{n}) = 0.5 \arctan\left(\frac{\int N_e(x,\hat{n}) 2B_s(x,\hat{n})B_t(x,\hat{n})dx}{\int N_e(x,\hat{n})[B_s^2(x,\hat{n}) - B_t^2(x,\hat{n})]dx}\right) \quad (1)$$

Where B_s is the component of the GMF perpendicular to the line of sight and B_t is the other GMF component perpendicular to the line of sight and to the Z-axis.
The obtained direction of the polarization angle is compared with the observational direction of polarized emission at 22 GHz which is given by equat

$$\gamma_{obs} = 0.5 \arctan\left(\frac{U(\hat{n})}{Q(\hat{n})}\right) \quad (2)$$

To compare the observed map of the direction of polarization and that obtained with expression (1), the method of maximum likelihood is used:

- First, for each considered model of GMF, the parameter space is explored by fitting the observed direction of the polarization angle. It is performed a χ^2 test for every model:

$$\chi^2 = \sum_i \frac{(\gamma_{obs} - \gamma_{model})_i^2}{\sigma_i^2}$$

The sum runs over the number of pixels we are considering.

- Second, the likelihood is defined as: $\mathcal{L} \propto e^{-0.5\chi^2}$

Once we get the maximum value for every model and every combination of parameters, the map is plotted for the best set of parameters of the best model (see figure 3).
For a given model, the marginal probability curve of a fixed parameter is obtained by integrating the likelihood for the others parameters.

Results

In table 1 it is shown the results obtained for the exploration of the parameter space.

Model	Parameters	Best fits	Min(χ^2)
Page et al.	ψ_0, ψ_1, χ_0	$65°, 4°, 16°$	91.04
Axisymmetric (ASS)	p,B_0	$27.4°, 6\mu G$	98.11
4-arm Bisymmetric	p,B_0	$27.3°, 5.5\mu G$	98.68
Concentric Ring	D_r,w,B_0	$0.6kpc, 8.1kpc, 6\mu G$	186.24
Turbulent Diffusion	k, σ_1,σ_2	$70\mu G, 10.01kpc, 10.5kpc$	192.74

- **Page et al. Model** : The obtained values of the parameters are (with a 95% confidence level):

$\psi_0 = 65° \pm^{6.42°}_{4.15°}$ and ψ_1 remains unconstrained with an upper limit of 36.95º.

$\chi_0 = 16° \pm^{15.38°}_{14.31°}$

The total number of maps for this exploration was 22509. The marginal probability distribution for these parameters is shown in figure 1.

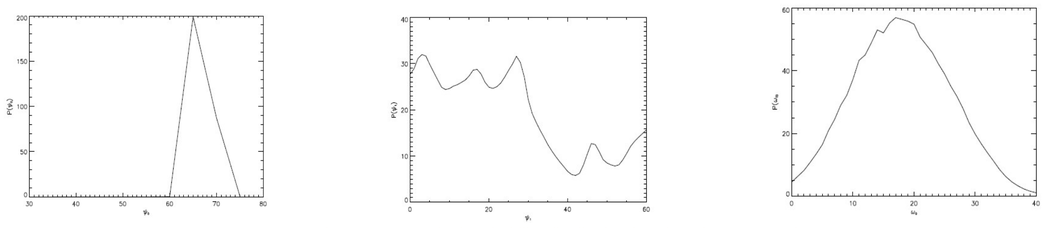

Figure 1. – Marginal probability distribution functions for ψ_0, ψ_1 and χ_0

- **Axisymmetric Model**: The obtained values of the parameters of this model are: B_0 = $(6\pm3)\mu G$ and $p = (27.4\pm^{1.7}_{3.9})°$

In this case the total number of maps was 700.

- **Bisymmetric Model**: The obtained values are $p = (27.3\pm^{1.6}_{4.0})°$ and $B_0 = (5.5 \pm 3)\mu G$

The total number of maps obtained in the exploration of the parameters space for this model was 1540.

- **Concentric Circular Ring**: The best fit reveal the following values: $D_r = (0.6\pm^{0.2}_{0.5})kpc$, $w = (8.1\pm^{9.5}_{6.4})kpc$ and $B_0 = (6 \pm 3)\mu G$

The total number of maps in this case, was 500.

- **Turbulent Diffusion Model**: The values of the parameters are: $K = (70\pm^{23.5}_{35.8})\mu G$, $\sigma_1 = 10.01\pm^{3.2}_{4.5})kpc$ and $\sigma_2 = 10.5\pm^{2.6}_{4.9})kpc$.

The total number of maps was 37516.

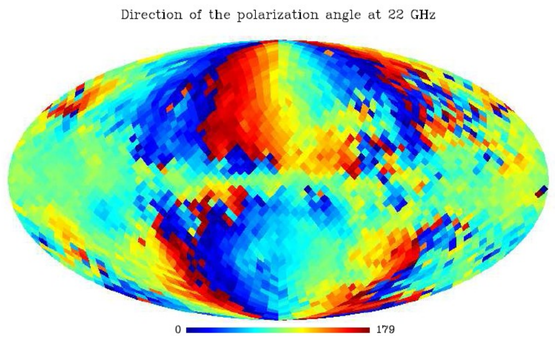

Figure 2.- Observed direction of the polarization angle from K-band.

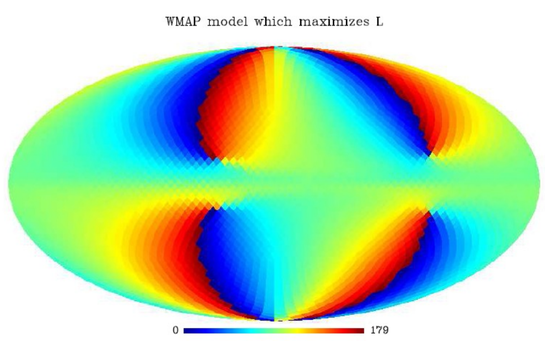

Figure 3.- Map of the direction of the polarization angle obtained with the Page et al. model that better fits the observed direction

Conclusions

The model that better reproduces the direction of the observed polarization angle of the synchrotron emission at 22 GHz is the m $\psi_0 = 65°\pm^{6.42°}_{4.15°}$, $\chi_0 = 16°\pm^{15.38°}_{14.31°}$ $\psi_0 = 35°$, $\psi_1 = 0.9°$ and $\chi_0 = 25°$(2007) with the parameters and ψ_1 unconstrained with an upper limit of 36.92º obtained with a 95% of confidence level. The values given by Page et al. (2007) were . As it is seen, ψ_0 is not compatible with the value obtained by Page et al. χ_0 is compatible taking into account the confidence limits.

For the Axisymmetric and Bisymmetric models it is concluded that the value obtained for the pitch angle is not compatible with the values derived from the observations of rotation measure of pulsars and extragalactic radio sources (p ~ [-5º,-15º]). For the Axisymmetric model, these results have been modified by assuming a radial variation of the amplitude of the magnetic field, B_0 in the paper we are preparing.

Concerning the Concentric Circular Ring model, we obtain values compatible with those obtained by Rand and Kulkarni (1989).

The method used here was very noisy for the Turbulent Diffusion model and did not produce available results. Reasonable values would be K ~ 0.23 μG, σ_1 ~ 0.01 kpc and σ_2 ~ 0.7 kpc.

We agree with the Page et al. model, however, we find clear discrepancies between the values of the parameters (particularly of ψ_0 and χ_0) given by Page et al and the ones obtained here. In general, there are small differences in the values of the χ^2 test for the comparison of the observed and calculated direction of the polarization angle. This fact implies that we cannot reject other models, being PLANCK measurements necessary to obtain firm constraints.

References

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