CHASING LAMBDA

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MODELS WITH DARK ENERGY

1 ΛCDM model $\quad H^2 = H_0^2\{\Omega_{\mathrm{m},0}(1+z)^3 + (1-\Omega_{\mathrm{m},0})\}$

$p_X = -\rho_X$ (Weinberg 1989)

2 Model with generalized Chaplygin gas $\quad H^2 = H_0^2\left\{\Omega_{\mathrm{m},0}(1+z)^3 + (1-\Omega_{\mathrm{m},0})[A_S + (1-A_S)(1+z)^{3(1+\alpha)}]^{\frac{1}{1+\alpha}}\right\}$

$p_X = -\frac{A}{\rho_X^\alpha}$ $\quad A > 0$ and $\alpha = \mathrm{const}$

3 Model with phantom dark energy $\quad H^2 = H_0^2\{\Omega_{\mathrm{m},0}(1+z)^3 + (1-\Omega_{\mathrm{m},0})(1+z)^{3(1+w_X)}\}$

$p_X = w_X\rho_X$ $\quad w_X < -1$ (Caldwell 2002)

4 Model with dynamical E.Q.S $\quad H^2 = H_0^2\left\{\Omega_{\mathrm{m},0}(1+z)^3 + (1-\Omega_{\mathrm{m},0})(1+z)^{3(w_0+w_1+1)}\exp[-\frac{3w_1 z}{1+z}]\right\}$

$w(a) = w_0 + w_1(1-a)$ (Chevallier & Polarski 2001)

5 Quintessence model $\quad H^2 = H_0^2\left\{\Omega_{\mathrm{m},0}(1+z)^3 + (1-\Omega_{\mathrm{m},0})(1+z)^{3(1+w_0(1+z)^{-\alpha})}\right\}$

$\rho_X = \rho_{X0}a^{-3(1+\bar{w}_X(a))}$ (Peebles & Ratra 1988)

$\bar{w}_X(a) = \frac{\int w_X(a)d\ln a}{\int d(\ln a)} \equiv w_0 a^\alpha$ (Rahvar & Movahed 2007)

mean of the coefficient of the EQS
in the log scale factor

MODELS WITH MODIFIED THEORY OF GRAVITY

6	DGP model	$H^2 = H_0^2 \left\{ \left[\sqrt{\Omega_{\mathbf{m},0}(1+z)^3 + \Omega_{rc,0}} + \sqrt{\Omega_{rc,0}} \right]^2 \right\}$	
	(Dvali et al. 2000)	$\Omega_{rc,0} = \frac{(1-\Omega_{\mathbf{m},0})^2}{4}$	
7	BΛCDM model	$H^2 = H_0^2 \left\{ \Omega_{\mathbf{m},0}(1+z)^3 - \Omega_{n,0}(1+z)^n + 1 - \Omega_{\mathbf{m},0} + \Omega_{n,0} \right\}$	(Singh & Vandersloot 2005)
8	interacting model with Λ	$H^2 = H_0^2 \{ \Omega_{\mathbf{m},0}(1+z)^3 + \Omega_{\mathbf{int},0}(1+z)^n + 1 - \Omega_{\mathbf{m},0} - \Omega_{\mathbf{int},0} \}$	(Szydlowski et al. 2006)
9	Cardassian model	$H^2 = H_0^2 \left\{ \Omega_{r,0}(1+z)^4 + \Omega_{\mathbf{m},0}(1+z)^4 \left[\frac{1}{1+z} + (1+z)^{-4+4n} \left(\frac{1-\Omega_{r,0}-\Omega_{\mathbf{m},0}}{\Omega_{\mathbf{m},0}} \right) \left(\frac{\frac{1}{1+z} + \frac{\Omega_{r,0}}{\Omega_{\mathbf{m},0}}}{1+\frac{\Omega_{r,0}}{\Omega_{\mathbf{m},0}}} \right)^n \right] \right\}$	
	$\Omega_{r,0} = 10^{-4}$; $3H^2 = \rho + B\rho^n$		(Freese & Lewis 2002)
10	Sahni-Shtanov brane I model	$H^2 = H_0^2 \left\{ \Omega_{\mathbf{m},0}(1+z)^3 + \Omega_{\sigma,0} + 2\Omega_{l,0} - 2\sqrt{\Omega_{l,0}}\sqrt{\Omega_{\mathbf{m},0}(1+z)^3 + \Omega_{\sigma,0} + \Omega_{l,0} + \Omega_{\Lambda b,0}} \right\}$	
	(Shtanov 2000)	$\Omega_{\sigma,0} = 1 - \Omega_{\mathbf{m},0} + 2\sqrt{\Omega_{l,0}}\sqrt{1 + \Omega_{\Lambda b,0}}$	

BAYESIAN FRAMEWORK OF MODEL SELECTION

The best one - the greatest value of the POSTERIOR PROBABILITY defined as

$$P(M_i|D) = \frac{P(D|M_i)P(M_i)}{P(D)}$$

$P(D)$ - normalization constant:

$$\sum_{i=1}^{K} P(M_i|D) = 1 \longrightarrow P(D) = \sum_{i=1}^{K} P(D|M_i)P(M_i)$$

- conclusions depend on the set of models under consideration

$P(M_i)$ - prior probability for i-th model

- value depends on our previous information (knowledge) about the models

BAYESIAN FRAMEWORK OF MODEL SELECTION

$P(D|M_i)$ - marginal likelihood (also called evidence):

$$P(D|M_i) = \int L(\bar{\theta}_i|D, M_i) P(\bar{\theta}_i|M_i) d\bar{\theta}_i \equiv E_i$$

likelihood of the model

vector of model parameters

prior probability for model parameters

approximation to -2 ln E →

$$BIC = -2 \ln \mathcal{L} + d \ln N$$

maximum likelihood

number of model parameters

number of data points

assumptions →

1. iid
2. $f(x_i|\bar{\theta}) = \exp\left[\sum_{k=1}^{S} w_k(\bar{\theta}) t_k(x_i) + b(\bar{\theta})\right]$
3. prior for model parameters ≠ 0 in the maximum likelihood
4. sample size large with respect to the number of model parameters

Schwarz 1978

1. prior for model parameters ≠ 0 in the neighborhood of the maximum likelihood
2. prior is bound in the whole parameter space
3. sample size large with respect to the number of model parameters

Cavanough & Neath 1999

BAYESIAN FRAMEWORK OF MODEL SELECTION

POSTERIOR ODDS

$$O_{12} \equiv \frac{P(M_1|D)}{P(M_2|D)} = \frac{P(M_1)}{P(M_2)} \frac{P(D|M_1)}{P(D|M_2)}.$$

BAYES FACTOR - B_{12}

$2 \ln B_{12}$	evidence in favor model 1
<0,2)	not worth than a bare mention
<2,6)	positive
<6,10)	strong
>10	very strong

(Kass & Raftery 1995)

APPLICATION TO COSMOLOGICAL MODELS COMPARISON

1. SN Ia: $N = 192$ - 60 ESSENCE; 57 SNLS; 30 HST; 45 local sample

(Riess et al. 2007; Wood-Vasey et al. 2007; Davis et al. 2007)

$$L_{SN} \propto \exp\left[-\frac{1}{2}\left(\sum_{i=1}^{N}\frac{(\mu_i^{theor}-\mu_i^{obs})^2}{\sigma_i^2}\right)\right],$$

$\mu_i^{obs} = m_i - M;$
$\mu_i^{theor} = 5\log_{10} D_{Li} - 5log_{10}H_0 + 25;$
$D_{Li} = H_0 d_{Li} = H_0(1+z_i)c\int_0^{z_i}\frac{dz'}{H(z')}.$

2. CMB

$$L_R \propto \exp\left[-\frac{(R^{\rm theor}-R^{\rm obs})^2}{2\sigma_R^2}\right],$$

$R^{\rm theor} = \sqrt{\Omega_{\rm m,0}}\int_0^{z_{dec}}\frac{H_0}{H(z)}dz,$

$R^{\rm obs} = 1.70 \pm 0.03 \quad z_{\rm dec} = 1089$

(Spergel et al. 2006; Wang & Mukherjee 2006)

APPLICATION TO COSMOLOGICAL MODELS COMPARISON

3. BAO

$$L_A \propto \exp\left[-\frac{(A^{\text{theor}} - A^{\text{obs}})^2}{2\sigma_A^2}\right],$$

$$A^{\text{theor}} = \sqrt{\Omega_{\text{m},0}} \left(\frac{H(z)}{H_0}\right)^{-\frac{1}{3}} \left[\frac{1}{z_A} \int_0^{z_A} \frac{H_0}{H(z)}\right]^{\frac{2}{3}},$$

$A^{obs} = 0.469 \pm 0.017$ $\quad z_A = 0.35$

(Eisenstein et al. 2005)

4. H(z): N=9 (differential ages $\left(\frac{dt}{dz}\right)$ of the passively evolving galaxies)

$$L_H \propto \exp\left(-\frac{1}{2}\left[\sum_{i=1}^{N} \frac{(H(z_i) - H_i(z_i))^2}{\sigma_i^2}\right]\right).$$

$$H(z) \equiv \frac{\dot{a}}{a} = -\frac{1}{1+z}\frac{dz}{dt}.$$

(Simon et al. 2005)

$$L \propto L_{sn} L_R L_A L_H$$

N=192+1+1+9

POSTERIOR PROBABILITIES FOR MODELS WITH DARK ENERGY

1	ΛCDM model	**0.84**
2	Model with generalized Chaplygin gas	0.02
3	Model with phantom dark energy	0.06
4	Model with dynamical E.Q.S	0.04
5	Quintessence model	0.04

POSTERIOR PROBABILITIES FOR MODELS WITH MODIFIED GRAVITY

6	DGP model	0.07
7	BΛCDM model	0.03
8	interacting model with Λ	0.13
9	Cardassian model $\Omega_{r,0} = 10^{-4}$	**0.74**
10	Sahni-Shtanov brane I model	0.03

POSTERIOR PROBABILITIES FOR ALL MODELS

1	ΛCDM model	**0.74**
2	Model with generalized Chaplygin gas	0.02
3	Model with phantom dark energy	0.05
4	Model with dynamical E.Q.S	0.04
5	Quintessence model	0.03
6	DGP model	0.01
7	BΛCDM model	0.005
8	interacting model with Λ	0.01
9	Cardassian model $\Omega_{r,0} = 10^{-4}$	0.09
10	Sahni-Shtanov brane I model	0.005

CONCLUSIONS

In the light of data used in analysis:

- ΛCDM model is the best one from the set of models with dark energy as well as the best one from the set of all models considered
- Cardassian model is the best one from the set of models with modified gravity